Computation of Flow Angles and Dynamic Pressure on BAT Probe

Richard M. Eckman

September 1999

Over the past couple of years, there has been some discussion of the best equations to use in `makepod` for computing the angle of attack α, angle of sideslip β, and the dynamic pressure q. Most of the discussion has centered on the three methods described in Leise and Masters (1991), which they call the low-resolution, high-resolution, and NCAR methods. All three methods start from the same basic equations for the pressure distribution over a sphere, and mainly differ in what pressures are assumed to be the independent (*i.e.*, measured) variables.

Early versions of `makepod` used the low-resolution method, which assumes that the measured variables are δp_y, δp_z, and q, in which δp_y is the pressure difference between the right and left ports on the probe and δp_z is the pressure difference between the bottom and top ports. The problem with this approach is that the BAT system actually measures the difference $\delta p_x = p_0 - p_r$ between the pressure p_0 at the central port and the pressure p_r obtained by averaging the pressure from the four "static" ports. Generally, δp_x will only equal q if α and β are zero, and p_r equals the static pressure p_s. To get around this problem, an iterative approach was used in which δp_x was assumed to be a first guess of q.

In the newer version of `makepod` (Eckman et al. 1999), the low-resolution method was replaced by the NCAR method. This method assumes the measured variables are δp_y, δp_z, and $p_0 - p_s$. The main assumption required in using this method is that the average pressure p_r from the "static" ports is equal to p_s. In the most recent versions of `makepod`, other contributors have made further changes in which α and β are computed using the NCAR method, but q is computed using an equation from the high-resolution method. This appears to be an attempt to get around the problem that p_r may not equal p_s.

The primary reason there has been some debate on the best equations to use in `makepod` is that none of the three methods described in Leise and Masters (1991) really matches the configuration of the BAT probe, in which the measured pressures are δp_x, δp_y, and δp_z. The aim of this note is to derive a set of equations for α, β, and q that more closely match the BAT configuration.

The starting point of the derivation is the potential-flow equation (Brown et al. 1983; Leise and Masters 1991)

$$p(\mathbf{n}) = p_s + \frac{q}{4}\left[9(\mathbf{N}\cdot\mathbf{n})^2 - 5\right] \tag{1}$$

for the pressure distribution over a sphere. Here, $\mathbf{N}$ is a unit direction vector that is normal to the sphere's surface at the stagnation point, $\mathbf{n}$ is the normal direction vector at some chosen point on the sphere, and $p(\mathbf{n})$ is the pressure at the chosen point. The unit vector $\mathbf{N}$ is equal to (Leise and Masters 1991)

$$\mathbf{N} = \frac{1}{D}\mathbf{i} - \frac{\tan\beta}{D}\mathbf{j} - \frac{\tan\alpha}{D}\mathbf{k}\,, \tag{2}$$

where $D = \sqrt{1+\tan^2\alpha+\tan^2\beta}$ and $(\mathbf{i},\mathbf{j},\mathbf{k})$ are the unit vectors for the probe coordinate system. The sign conventions in Eq. (2) have been altered from Leise and Masters (1991) to match the coordinate system used with the BAT system.

Equations for δp_x, δp_y, and δp_z can be derived from Eq. (1) by determining the values of $\mathbf{n}$ that correspond to the positions of the various ports on the probe. For δp_y, and δp_z, it is assumed that the right-left and top-bottom ports are at an angle ϕ away from the center port. Equation (1) then gives

$$\delta p_y = 9q\frac{\sin\phi\cos\phi}{D^2}\tan\beta\,; \tag{3}$$

$$\delta p_z = 9q\frac{\sin\phi\cos\phi}{D^2}\tan\alpha\,. \tag{4}$$

To compute δp_x, an equation must first be derived for the reference pressure p_r. It is assumed that p_r represents the average pressure of the four "static" pressure ports. (Some calculations performed using pipe-flow theory suggest that this is a good assumption as long as the tubes connecting the four ports are of the same length.) If the "static" ports are at an angle ϕ_r from the center port, then p_r is found to be

$$p_r = p_s + \frac{q}{4}\left[9\frac{\cos^2\phi_r + 0.5\,(\tan^2\alpha + \tan^2\beta)\sin^2\phi_r}{D^2} - 5\right] . \tag{5}$$

The appropriate equation for $\delta p_x = p_0 - p_r$ is then

$$\delta p_x = \frac{9q}{8D^2}\sin^2\phi_r\left[3 - D^2\right] . \tag{6}$$

Equations (3), (4), and (6) form a closed system of equations that can be solved for the unknowns α, β, and q. To solve this system, it is is useful to define the ratios

$$H_y = \frac{\sin^2\phi_r}{8\sin\phi\cos\phi}\,\frac{\delta p_y}{\delta p_x} ; \tag{7}$$

$$H_z = \frac{\sin^2\phi_r}{8\sin\phi\cos\phi}\,\frac{\delta p_z}{\delta p_x} . \tag{8}$$

The flow angles can then be obtained as

$$\tan\alpha = \frac{4H_y}{1 + \sqrt{1 + 8\left(H_y^2 + H_z^2\right)}} ; \tag{9}$$

$$\tan\beta = \frac{4H_z}{1 + \sqrt{1 + 8\left(H_y^2 + H_z^2\right)}} , \tag{10}$$

and q as

$$q = \frac{8}{9}\,\frac{\delta p_x}{\sin^2\phi_r}\,\frac{1 + \tan^2\alpha + \tan^2\beta}{2 - \tan^2\alpha - \tan^2\beta} . \tag{11}$$

Equations (9)–(11) are different from any of the three methods described by Leise and Masters (1991). However, they more closely match the pressure variables that are available from the BAT system. Note also that the BAT system has a separate channel for the reference pressure p_r. This means that the static pressure p_s can be estimated from Eq. (5) once α, β, and q are computed.

In the equations derived above, the positions ϕ and ϕ_r of the pressure ports have been retained as known constants. Theoretically, an angle of 41.81° is useful for ϕ_r, because p_r will then equal p_s when $\alpha = \beta = 0$. (But not for other flow angles.) For ϕ, an angle of 45° has often been used. One of the early pressure spheres used on the Long-EZ aircraft apparently did use $\phi_r = 41.81°$ and $\phi = 45°$. However, wind-tunnel tests of this sphere indicated that $p_r \neq p_s$ even when the stagnation point was at the center port. This was interpreted as resulting from deviations of the real-world flow from the potential-flow theory given in Eq. (1). As a result, the "static" ports were moved back to 45°, which is why the current BAT system uses $\phi = \phi_r = 45°$.

The movement of the "static" ports to 45° brought p_r closer to p_s, but this leads to some confusion regarding the application of Eqs. (7)–(11). Should these be applied using the original angles of $\phi_r = 41.81°$ and $\phi = 45°$, or using the corrected angles $\phi = \phi_r = 45°$? In the past, the original angles have usually been used, but it is not clear that this is the best approach. The issue is difficult to resolve, because the movement of ϕ_r from 41.81° to 45° was done to counter viscosity effects which are totally ignored in the derivations of Eqs. (7)–(11).

References

Brown, E. N., C. A. Friehe, and D. H. Lenschow, 1983: The use of pressure fluctuations on the nose of an aircraft for measuring air motion. *J. Climate Appl. Meteor.*, **22**, 171–180.

Eckman, R. M., T. L. Crawford, E. J. Dumas, and K. R. Birdwell, 1999: Airborne meteorological measurements collected during the Model Validation Program (MVP) field experiments at Cape Canaveral, Florida, in press as a NOAA Technical Memorandum, NOAA Air Resources Laboratory, Atmospheric Turbulence and Diffusion Division, Oak Ridge, TN.

Leise, J. A. and J. M. Masters, 1991: Wind measurement from aircraft, draft report, NOAA Aircraft Operations Center, Miami, FL.